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ORIGINAL PAPER

A review of the formulation and application of the spatial equilibrium models to analyze policy

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Abstract: This paper reviews alternative market equilibrium models for policy analysis. The origin of spatial equilibrium models and their application to wood and wood-processing industries are described. Three mathematical programming models commonly applied to solve spatial problems - namely linear programming, non-linear programming and mixed complementary programming - are reviewed in terms of forms of objective functions and constraint equalities and inequalities. These programming are illustrated with numerical examples. Linear programming is only applied in transportation problems to solve quantities transported between regions when quantities supplied and demanded in each region are already known. It is argued that linear programming can be applied in broader context to transportation problems where supply and demand quantities are unknown and are linear. In this context, linear programming is seen as a more convenient method for modelers because it has a simpler objective function and does not require as strict conditions, for instance the equal numbers of variables and equations required in mixed complementary programming. Finally, some critical insights are provided on the interpretation of optimal solutions generated by solving spatial equilibrium models.

Keywords: simplex method, reduced-gradient, linear programming, non-linear programming, mixed complementary programming

Introduction

Many theoretical economic models have been designed to analyze policy, for instance partial equilibrium, general equilibrium, dynamic general equilibrium, spatial equilibrium, optimal control and dynamic spatial equilibrium models. The need for these models was raised in the late 19th century (Benjamin, 2005), although they were only fully developed after 1950. The general equilibrium model was first developed by Arrow, Debreu and McKenzie in the 1950s (Debreu 1983). The spatial equilibrium model was developed by Takayama et al. initially in 1964 and fully in 1973 (McCarl et al., 2002). The optimal control model was developed independently by Bellman in 1957 and Pontryagin et al. in 1962 (Weber, nd.)

Partial equilibrium models concern only one product and one region. General equilibrium models accommodate many products and one region. Spatial equilibrium models allow many products and many regions. Optimal control models have one product and one region in a given time period. Dynamic general equilibrium models accommodate many products and one region in a given period of time. Dynamic spatial equilibrium models have many products and many regions in a given period of time. Generally, each model is used to find equilibrium (optimal) points (e.g. prices, quantities supplied, quantities demanded, quantities imported and quantities exported) by maximizing its objective function (e.g. the total economic surplus) subject to its constraint equalities and inequalities (e.g. supply prices are equal to demand prices).

Policy changes usually alter directly the values of exogenous variables (e.g. elasticities of supply and demand) which in turn lead to changes in the values of the endogenous variables (e.g. prices). The changes of endogenous variables move the economic system to new equilibrium points. The change of total economic surplus between old and new equilibrium points reveals which policies are superior.

The next sections describe the overall structure of the various types of industry policy models and the relationships between these models. Reasons why particular theoretical models may be most suitable for specific products are then identified. The alternative mathematical programming techniques used to solve spatial equilibrium models and their advantages and disadvantages are discussed. Finally, the relationship between chosen functional forms of supply and demand and the interpretation of op-

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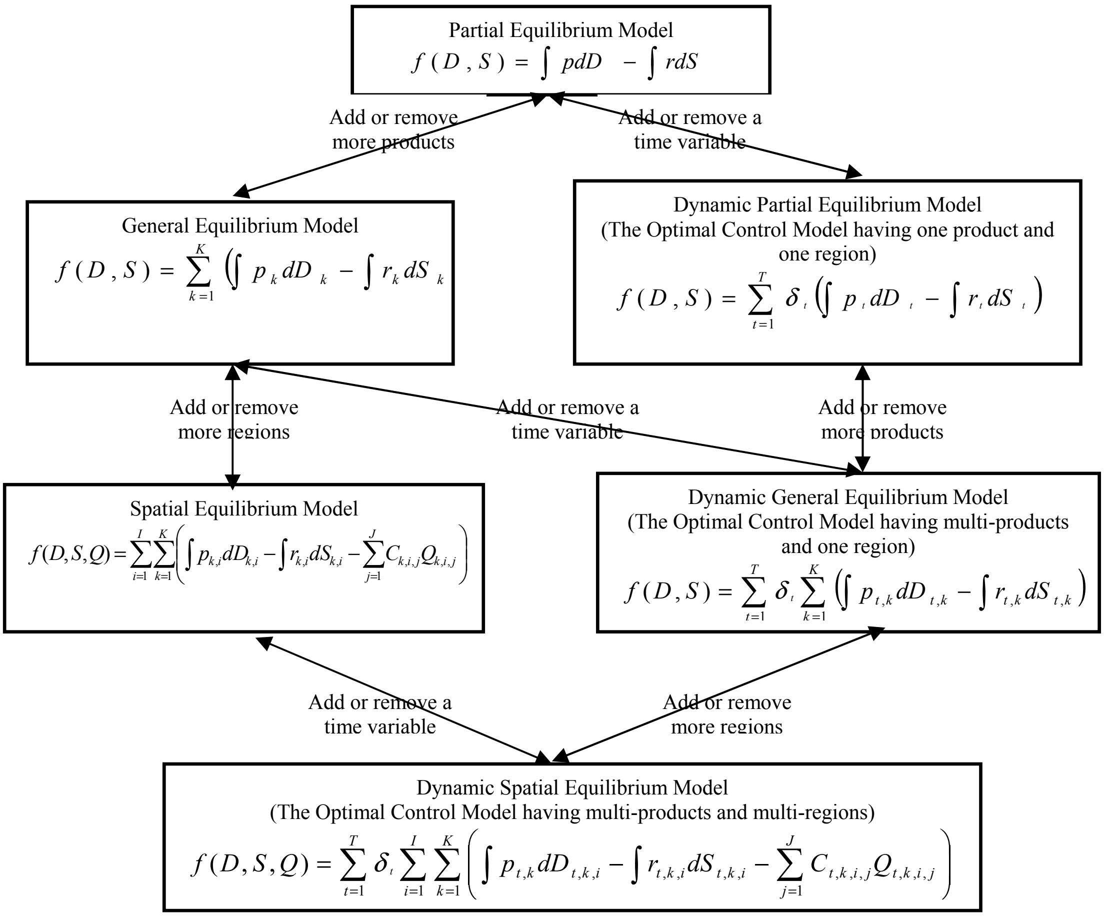
timal solutions generated are examined. These descriptions will slightly emphasize on the author's research field namely timber and timber-processing industries.

Alternative theoretical models and their application in policy analysis

The researcher is often faced with making a choice between partial equilibrium models, general equilibrium models, dynamic general equilibrium models, spatial equilibrium models, dynamic spatial equilibrium models, and optimal control models. The relationship between these models' formulations is presented via their objective functions, as illustrated in Fig. 1. For example in partial equilibrium models, supply and demand functions express supply and demand prices as functions of supply and demand quantities, $r = f(S_k)$ and $p = g(D_k)$ respectively. The integral of the demand function ($\int pdD$) is the area under a demand curve. This area is represented as the consumer's economic value. The integral of the supply function ($\int rdS$) is the area under the supply curve. This area is presented as the producer's production cost. The value of ($\int pdD - \int rdS$) is interpreted as the total economic surplus that is the sum of the consumer surplus and the producer surplus. As models become more complicated (e.g. moving from a partial equilibrium model to a dynamic spatial equilibrium model), supply and demand functions are expressed in more detail with time (t), product (k) and region (i) subscripts, for instance $r_{t,k,i} = f(S_{t,k,i})$ and $P_{t,k,i} = g(D_{t,k,i})$ for supply and demand respectively.



Definitions of symbols

i	region i, i = 1,2,…….., R.	D	demand quantity
j	region j, j = 1,2,…….., R.	S	supply quantity
K	product k, k = 1,2,……., K.	p	demand price
Q	transportation quantity between regions	r	supply price
C	unit transportation costs between regions	δ	discount factor
		t	discrete time points

Fig. 1 Differences between market equilibrium models expressed by objective functions

In terms of applications, the theoretical partial equilibrium models apply a high level of simplification of reality, with prices and quantities of other goods in other regions assumed not to interact with those in the analyzed market. Therefore, partial

equilibrium modelling is rarely applied for policy analysis. General equilibrium models are applied more widely for various products because these models can reflect the real economy of a country more exactly where all supply and demand of goods and services always interact to form a set of equilibrium points. For example, Jakfar (2007) applied a general equilibrium model for the plywood sector in Indonesia.

Of these alternative model types, spatial equilibrium models are applied much more widely in industry policy research. The main reason is spatial equilibrium models have all features of general equilibrium models and one more useful characteristic about analysing factors and policies affecting quantities of goods and services traded between regions and countries in the world. This characteristic is more appropriate for the world economy where trade liberalisation becomes more and more acceptable since 1980s. Example studies applying the spatial equilibrium models include Goletti et al. (1995) for the rice sector in Vietnam, Tsunemasa et al. (1997) spatial equilibrium model for 'imperfectly competitive milk markets' in the Japanese dairy industry, and Agriculture Consultant International (ACI, 2002) for livestock sector in Vietnam. Timber industry applications of the spatial equilibrium models include those of Adams et al. (1980) for timber in north America, Jae (1984) for the southern pine lumber in the USA, Yoshimoto et al. (2002) for Japanese timber markets, Stennes et al. (2005) for lumber in the USA and Canada, and Devadoss et al. (2005) for softwood lumber in the USA, Canada, Mexico, China, Japan, New Zealand, Australia and European Union.

The optimal control model is usually applied in disciplines where time affects strongly the total economic surplus, including financial economics and natural resource economics, including forestry economics. For example, Hassan et al. (2009) adopted an optimal control model to identify equilibrium points which balance wood consumption, harvesting rates and reforestation rates over time to maximise total economic surplus, in Sudan.

A literature search indicates that the dynamic spatial equilibrium model has not been applied to policy analysis, including wood and wood-processing industries. One reason may be that there is not a practical case where the prices of all products are highly correlated between many points of time and between all market locations. For example, a time variable is more important for farmers who plant timber trees than for wood-processing factories because the price of timber depends strongly on the ages of timber trees but the price of paper (an internationally traded commodity) changes only slowly over time. A distance variable is seen as more important for wood-processing factories than for farmers because the locations of planted production forest are largely fixed whereas the locations of factories can be flexible. Therefore, if a dynamic spatial equilibrium model for wood and wood-processing products is developed, the model will become unnecessarily large. Instead of applying a dynamic spatial equilibrium model, two much smaller models can be applied. For example, an optimal control model can be applied for only timber in one region and a spatial equilibrium model for paper in many regions. Sohngen (1998) reviewed four timber market models and explained why market predictions (e.g. prices) were different between the models, namely the Timber Assessment Market Model, the Global Trade Model, the Timber Supply Model and the North American Pulp and Paper Model. He explained that computational constraints limited the development of dynamic spatial equilibrium models by forest economists, stating 'In fact, the two theories could be merged to develop a spatial, dynamic optimization model, but forestry economists have not done this on the large scale yet (probably because of the computational problems associated with solving such a model*)*' Sohngen (1998, p. 6).

Steps required to construct spatial equilibrium models and analyze policy

The studies of Adams et al. (1980), Jae (1984), Yoshimoto et al. (2002), Stennes et al. (2005) and Devadoss et al. (2005) were all inspired by the idea of the spatial equilibrium model created by Takayama et al. in 1964. The study of Stennes et al. (2005) provides a particularly clear presentation of the methodology, dataset and steps to develop a spatial equilibrium model. The following nine steps were used to develop the spatial equilibrium model in their study.

1. Estimation of the price elasticities of supply and demand for all products and regions using econometric methods.
2. Collection of data in the base year of production, consumption, price and unit transportation cost for all products and regions included their model.
3. Estimation of constants of supply and demand for all products and regions, based on the estimated price elasticities in the first step and the data collected in the second step.
4. Solving of the spatial equilibrium model to find the optimal solution in the base year.
5. Comparing the optimal solutions obtained and the data of production, consumption and prices collected in step 2. If the generated optimal solutions in the base year are similar to the data collected in the base year, the spatial equilibrium model is ready for the policy analysis.
6. Setting up policy scenarios. Each policy scenario may contain a policy variable or a group of policy variables. For instance a scenario A includes a new import tax on sawn-wood imported of 5% in a current year instead of 30% in the base year.
7. Solving the model for each policy scenario and generating corresponding optimal solutions including total economic surplus, prices, quantities supplied, quantities demanded and quantities traded of all products.
8. Comparison of generated optimal solutions between policy scenarios.
9. Drawing conclusions and policy implications about prices, quantities supplied, quantities demanded, quantities traded and economic surplus.

Mathematical programming applied in spatial equilibrium models

Mathematical programming is used in the four main market equilibrium models namely partial equilibrium model, general equi-

librium model, spatial equilibrium model and optimal control model to identify equilibrium points (e.g. quantities supplied, demanded and traded, and product prices) by optimising the value of an objective function (e.g. maximising the total economic surplus) and satisfying all constraint equalities and inequalities (e.g. supply price is equal to demand price).

Linear programming is used to solve problems where the objective function, constraint equalities and inequalities are all linear. The simplex method, developed independently by Kantorovich in 1939 and Dantzig in 1948, is usually adopted (Chiang 1974; McCarl et al. 2002). Various software packages can be used for linear programming, for example the Solver facility in Microsoft Excel (for relatively small models) and the General Algebraic Modeling System (GAMS) software which will solve models with tens of thousands of activities and constraints. In the spatial analysis, linear programming is usually or commonly applied to solve transportation problems where quantities demanded and supplied in each region are already known. In this case, the optimal quantities traded between regions are obtained by minimizing the total transportation costs, as presented in McCarl et al. (2002). Moreover, linear programming can be applied to solve transportation problems where quantities demanded and supplied in each region are unknown and are linear functions. In this case, the objective function is still the function of the total transportation cost. The two known constraint levels of quantities supplied and quantities demanded are replaced by the supply and demand functions, as presented in Figure II.

Non-linear programming is used to solve problems where there is at least one non-linear function that can be the objective function or any constraint equality or inequality (Chiang 1974; McCarl et al. 2002). The gradient method developed by Lasdon et al. as reported in McCarl et al. (2002) – the so-called reduced-gradient method or gradient projection method or constraint derivatives or generalized reduced gradient – is usually applied to solve non-linear programming problems. The main principle of the gradient method is to find optimal solution where the increases or decreases in levels of slack variables do not affect the value of an objective function. In another words, the derivatives of the objective function with respect to slack variables are equal to zero. Like linear programming, non-linear programming is available in the Solver facility in Microsoft Excel and in the General Algebraic Modeling System (GAMS) software.[1] Non-linear programming is commonly used to solve spatial equilibrium models including the models that have linear functions of supply, demand and constraint equalities and inequalities, as presented in McCarl et al. (2002), Stennes et al. (2005) and Devadoss et al. (2005). In this case, the objective function of the total economic surplus is a quadratic function, as presented in Fig. 2. This problem is also called a quadratic programming problem.

Similar to non-linear programming, mixed complementary programming (MCP) is used to derive optimal solutions of market equilibrium models involving various forms of supply and demand functions (e.g. linear and logarithmic). As illustrated in Figure II, unlike linear and non-linear programming, the application of MCP does not require modelers to establish an objective function. However, MCP can only be applied in models where there are exactly as many variables as there are equations and each variable can be specified as being complementary with one and only one equation (McCarl et al. 2002). The condition where the number of constraints is equal to the number of variables also exists in some spatial equilibrium models. An example is research of ACI (2002) about applying a spatial equilibrium model to analyze policy for the livestock sector in Vietnam.

Successive quadratic programming can also be used to solve quadratic problems. The method of successive quadratic programming replaces quadratic objective functions by linear objective functions that are the sum of artificial variables. These artificial variables are added to the equations of the first order conditions (derivatives) of the Lagrange function with respect to price variables or quantity variables in the cases using inverse functions or original functions respectively. These equations are set to be equal to zero. Therefore, the quadratic models become linear models that can be solved with linear programming. However, successive quadratic programming is not available in the Solver facility in Microsoft Excel or in the GAMS software. Therefore, optimal solutions generated by using successive quadratic programming cannot be examined directly by using Microsoft Excel and GAMS software.[2]

Functional forms and interpretation of optimal solutions

Market equilibrium models in general and spatial equilibrium model in particular require various assumptions. For instance, it is assumed that markets are perfectly competitive markets and there is only one functional form of supply and demand. Theoretically, the functional forms differ between products and regions. However, in practice economists typically use one functional form – usually linear or logarithmic – for all equations in the spatial equilibrium model. For example, logarithmic functions were applied for all supply and demand equations in Goletti et al. (1995) and ACI (2002). Linear functions were adopted by

[1] Mathematicians have developed many solution algorithms to solve non-linear problems, for example, CONOPT, BARON, COINIPOPT, MINOS, MOSEK and OQNLP. Each facility is most suitable for solving a specific model formulation. Linear and logarithmic functions are common in the four market equilibrium models described in this paper.

[2] Compared to other methods, successive quadratic programming requires more modelers' effort to establish a linear objective function before applying linear programming installed in Excel or in GAMS to solve quadratic problems. There are two notable efforts. One is to derivative the Lagrange function by hand with respect to prices or quantities if original or inverse supply and demand functions are applied respectively. In other methods, this effort is usually done automatically by computer. Another is to add artificial variables in derivative equations. In other methods, this effort is not needed.

Takayama *et al.* (1964) did not present exactly the name of the mathematical method to solve the numerical examples in their paper. Rather, they reported briefly in a footnote that they replace the quadratic objective function ($f(p,r) = \int Ddp - \int Sdr$) by a new linear objective function ($- \Sigma\Sigma\, x_{i,j}v_{i,j}$). In the new objective function, $x_{i,j}$ are the quantities transported between regions i and j. Notably, Takayama *et al.* (p. 86) also included artificial variables ($v_{i,j}$). After that, they applied the simplex method to derive the optimal solutions of equilibrium prices, quantities supplied, quantities demanded and quantities traded between regions.

Stennes et al. (2005) and Devadoss et al. (2005). There is one notable reason leading to the application of only one functional form. The application makes the model simpler and hence a specific solver facility in a specific software package can be used conveniently for deriving optimal solutions.

Spatial equilibrium model with linear supply and demand functions or with unknown quantities supplied and demanded			Transportation model with known (fixed) quantities supplied and demanded	Explanation
Non-linear programming $f(D,S,Q)=\sum_{i=1}^{I}\sum_{k=1}^{K}\left(\int p_{k,i}dD_{k,i}-\int r_{k,i}dS_{k,i}-\sum_{j=1}^{J}C_{k,i,j}Q_{k,i,j}\right)$	Mixed complementary programming	Linear programming $f(Q)=\sum_{k=1}^{K}\sum_{i=1}^{R}\sum_{j=1}^{R}C_{k,i,j}Q_{k,i,j}$		
Constraint inequalities and equalities				
$r_{k,i} - r_{k,j} \le C_{i,j}$	$r_{k,i} - r_{k,j} \le C_{i,j}$	$r_{k,i} - r_{k,j} \le C_{i,j}$		The gaps of supply prices must be smaller than the unit transportation costs (t) by commodities (k) and regions (i)
$p_{k,i} - p_{k,j} \le C_{i,j}$	$p_{k,i} - p_{k,j} \le C_{i,j}$	$p_{k,i} - p_{k,j} \le C_{i,j}$		Supply price and demand price must be equal by commodities (k) and regions (i)
$p_{k,i}=\alpha_{k,i}-\sum_{k'=1}^{K}\beta_{k,k',i}D_{k',i}$	$p_{k,i}=\alpha_{k,i}-\sum_{k'=1}^{K}\beta_{k,k',i}D_{k',i}$	$p_{k,i}=\alpha_{k,i}-\sum_{k'=1}^{K}\beta_{k,k',i}D_{k',i}$		Demand equations
$r_{k,i}=\theta_{k,i}-\sum_{k'=1}^{K}\varpi_{k,k',i}S_{k',i}$	$r_{k,i}=\theta_{k,i}-\sum_{k'=1}^{K}\varpi_{k,k',i}S_{k',i}$	$r_{k,i}=\theta_{k,i}-\sum_{k'=1}^{K}\varpi_{k,k',i}S_{k',i}$		Supply equations
$\sum_{j=1}^{R}Q_{k,i,j}=S_{k,i}$		$\sum_{j=1}^{R}Q_{k,i,j}=S_{k,i}$	$\sum_{j=1}^{R}Q_{k,i,j}=S^{0}_{k,i}$	Total quantity transport of a commodity k from a region i to all regions j must be equal to total supply of a commodity k in region i
$\sum_{i=1}^{R}Q_{k,i,j}=D_{k,j}$		$\sum_{i=1}^{R}Q_{k,i,j}=D_{k,j}$	$\sum_{i=1}^{R}Q_{k,i,j}=D^{0}_{k,j}$	Total quantity transport of a commodity k from all regions i to a regions j must be equal to total demand of a commodity k in region j
	$S_{k,i}+\sum_{j=1}^{R}Q_{k,j,i}-\sum_{j=1}^{R}Q_{k,i,j}=$			Quantity supplied plus total quantities exported to region j subtract total quantity imported from region j must be equal to quantity demanded for a commodity k in region i.
		$\sum_{i=1}^{R}S_{k,i}=\sum_{i=1}^{R}D_{k,i}$		Total supply of a commodity k from all regions must be equal to the total demand of a commodity k from all regions.
$p_{k,i}, r_{k,i}, Q_{k,i,j} \ge 0$	$p_{k,i}, r_{k,i}, Q_{k,i,j} \ge 0$	$p_{k,i}, r_{k,i}, Q_{k,i,j} \ge 0$	$Q_{k,i,j} \ge 0$	All variables (supply prices, demand prices and quantity transported among regions) are non-negative.

Fig. 2 Differences between models applied NLP, LP and MCP expressed through objective functions, constraint equalities and inequalities

The application of only one functional form for all supplies, demands and constraints can lead to the situation where an optimal solution can be found mathematically but the solution does not have accepted economic meaning. Fig. 3 illustrates this situation. The assumed linear supply curve is only acceptable for generated optimal prices that are higher than the point of the lowest average production cost. However, an optimal solution can be found mathematically and may include an optimal price below the point of the lowest production cost. If the modeler does not realize this problem, their interpretation of the optimal solution will be incorrect. To avoid this situation, the application of only one functional form needs support from other research

findings (e.g. econometric research) to indicate the range of points over which the functions are nearly linear.

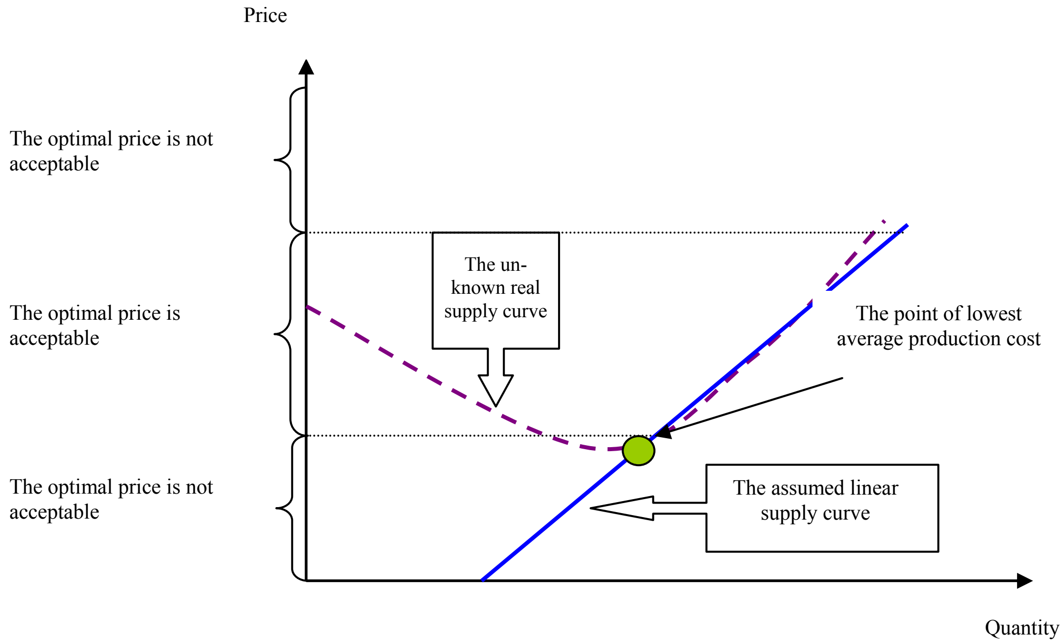


Fig. 3 An example of the relationship between the real unknown supply function and an assumed linear supply function

The further interpretation issue relates to the relationship between functional forms of supply and demand and the interpretation of the total economic surplus, i.e. the sum of the consumer and producer surplus. Mathematically, the behaviour of producers is expressed as the supply quantity being a function of supply price, $S = f(r)$, called the original supply function. Similarly, the behaviour of producers is expressed the supply price being a function of supply quantity, $r = f(S)$, called the inverse supply function. The behaviour of buyers can also be expressed in two alternative ways, $D = f(P)$ or $P = f(D)$. The inverse functions $r = f(S)$ and $P = f(D)$ are applied in most economics textbooks and mathematical programming books and international journals, for instance a economics book of Mansfield (1996) and journals of Stennes et al. (2005) and Devadoss et al. (2005). In these cases, the objective function of $\int PdD - \int rdS$ is represented as the total economic surplus (CS+PS), as presented Fig. 4. In some other studies, for instance ACI (2002) and numerical examples presented in Takayama et al. (1964), the original functions of $S = f(r)$ and $D = f(P)$ are applied. In this context, the objective function of $\int Ddp - \int Sdr$ is the total loss including the producer's production cost (PC) and the consumer loss (CL), as illustrated in Fig. 4.

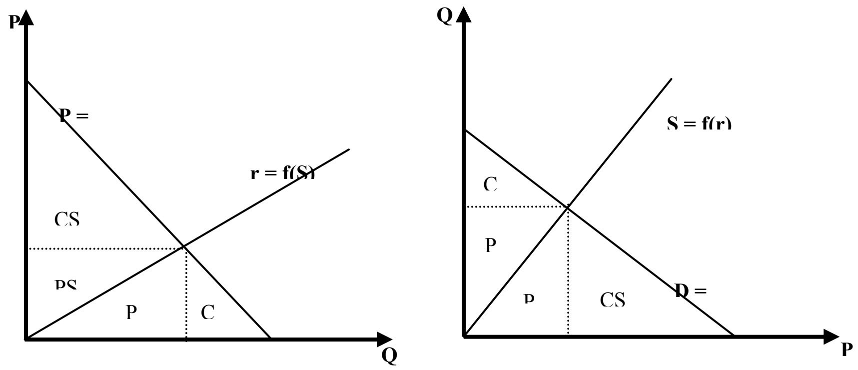


Fig. 4. The positions of CS, PS, PC and CL according to the applications of inversed functions on the left and original function on the right

The applications of the objective function $\int PdD - \int rdS$ or $\int Ddp - \int Sdr$ generate similar optimal solutions of quantities supplied, demanded and traded and of prices. However, they differ in terms of interpreting the values of objective functions. Therefore, modelers who apply the original functions should be careful when they report the values of chosen objective functions. With the assistance command syntaxes, for instance in General Algebraic Modeling System (GAMS), the total economic surplus ($\int PdD - \int rdS$) can be rather easily calculated

from the known total loss ($\int Ddp - \int Sdr$).

Mathematical programming applied for numerical examples

The optimal solutions of spatial equilibrium model with linear supply and demand functions can be solved by any of the three mathematical methods, namely linear programming, non-linear programming and mixed complementary programming (NLP, LP and MCP). The optimal solutions include equilibrium prices, quantities supplied, quantities demanded and quantities traded between regions. An example below is taken and slightly modified from Takayama et al. (1964) (The program file for this example written in GAMS was published in GAMS Corporation's website (http://www.gams.com/modlib/libhtml/spatequ.htm) on November 2010.)

Example, Hypothetical supply and demand functions for two products in three regions are presented in Table 1.

Table 1. Linear supply and demand functions for two products in three regions

Function	Region		
	Region 1	Region 2	Region 3
Product 1			
$D_{ki} = \alpha_{ki} - \sum_{k'=1}^{K} \beta_{kk',i} P_{k'i}$	$D_{11} = 200\text{-}10p^d_{11}+p^d_{12}$	$D_{21} = 100 - 5p^d_{21} + p^d_{22}$	$D_{31} = 160 - 8p^d_{31} + p^d_{32}$
$S_{ki} = \theta_{ki} - \sum_{k'=1}^{K} \varpi_{kk',i} r_{k'i}$	$S_{11} = -50+10p^s_{11}+0.5p^s_{12}$	$S_{21} = -50+20p^s_{21}+0.5p^s_{22}$	$S_{31} = -50+10p^s_{31}+0.5p^s_{32}$
Product 2			
$D_{ki} = \alpha_{ki} - \sum_{k'=1}^{K} \beta_{kk',i} P_{k'i}$	$D_{12} = 300+ p^d_{11} -10p^d_{12}$	$D_{22} = 200+p^d_{21} -20p^d_{22}$	$D_{32} = 250+p^d_{31} -10p^d_{32}$
$S_{ki} = \theta_{ki} - \sum_{k'=1}^{K} \varpi_{kk',i} r_{k'i}$	$S_{12} = -60+0.5p^s_{11}+15p^s_{12}$	$S_{22} = -60+ 0.5p^s_{21}+ 25p^s_{22}$	$S_{31} = -60+0.5p^s_{31}+15p^s_{32}$

Note: *r* is subscript for region (*r* = 1,2,3) and i is subscript for product (*i* = 1,2). The numerical value of the constant in the demand function of product 2 in region 2 created by Takayama *et al.* (1964) was 150. The number is the constant of the demand function for product 2 in region 2. Takayama *et al.* reported their optimal solutions generated by solving their spatial equilibrium model with the figure of 150. However, they did not recognize that their optimal solutions were inconsistent with this figure. The demand price and supply price for product 2 in region 2 differ from each other; for more detail see Takayama *et al.* (1964, p. 88). To be consistent, the author changed this figure of 150 to 200.

The unit transportation costs ($t_{i,r,r'}$, the cost to transport a product *i* from region r to region r') are as in Table 2.

Table 2. Unit transportation cost between regions

Source region	Destination region		
	Region 1	Region 2	Region 3
Product 1			
Region 1	0	2	2
Region 2	2	0	1
Region 3	2	1	0
Product 2			
Region 1	0	3	3
Region 2	3	0	2
Region 3	3	2	0

Again suppose that an economist would like to find answers for four questions, namely: (1) What are the equilibrium prices (supply price, P^s_{ri}, equal to demand price, P^d_{ri}) in each three regions?, (2) What are optimal quantities supplied (S_{ri}) in each three regions, (3) What are optimal quantities demanded (D_{ri}) in each three regions? and (4) What are optimal quantities transported ($Q_{rr',i}$) between regions?.

Optimal solutions were obtained by using MCP, non-linear and linear CONOPT Solver facilities in GAMS. The optimal solutions as presented in Tables 3 and 4 are identical between the three mathematical methods.

Table 3. Optimal solutions of prices, quantities supplied and demanded - output of the computer program

Region	Equilibrium Price		Quantity demanded		Quantity supplied	
	C1	C 2	C1	C2	C1	C2
Region 1	11.00	11.64	101.00	195.27	65.50	120.36
Region 2	9.00	8.64	64.00	35.91	134.50	160.23
Region 3	10.00	10.64	90.00	154.27	55.00	104.86

Note: C1---Commodity 1; C2--- Commodity 2

Table 4. Optimal solutions of quantities transported between regions - output of the computer program

From region r	To Region r'	Quantity transported	
		Commodity 1	Commodity 2
Region 1	Region 1	65.50	120.36
Region 2	Region 1	35.50	74.91
Region 2	Region 2	64.00	35.91
Region 2	Region 3	35.00	49.41
Region 3	Region 3	55.00	104.86

The optimal solutions in Tables 3 and 4 indicate the equilibrium prices of commodity 1 are 11, 9 and 10 in regions 1, 2 and 3 respectively. Those of commodity 2 are 11.64, 8.64 and 10.64

in regions 1, 2 and 3 respectively. At these equilibrium prices, region 2 is the only surplus region. The quantities supplied of the commodity 1 (134.50) and the commodity 2 (160.23 are higher than the quantities demanded of the commodity 1 (64.00) and the commodity 2 (35.91). Its quantities exported of the commodity 1 are 35.50 to region 1 and 35.00 to region 3. Its quantities exported of the commodity 2 are 74.91 to region 1 and 49.41 to region 3.

As demonstrated in these two examples of spatial equilibrium models, optimal solutions can be found and are exactly equal to each other when LP, NLP and MCP are applied.

Concluding comments

Non-linear programming and mixed complementary programming can be used to solve market equilibrium models with various types of supply and demand functions. In the case of spatial equilibrium models with linear functions of supply and demand, non-linear programming is applied and the objective function of total economic surplus is a quadratic function. Unlike non-linear programming, the application of mixed complementary programming does not require modelers to establish any objective function. However, mixed complementary programming can only be applied under the strict condition that there are exactly as many variables as there are equations and each variable can be specified as being complementary with one and only one equation. The review of literature demonstrates that linear programming is only applied in transportation problems to solve quantities transported between regions when quantities supplied and demanded in each region are known. This paper argues and demonstrates that linear programming can be applied in a broader context to transportation problems where quantities supplied and demanded in each region are unknown and are linear functions of prices. In this context, compared to non-linear and mixed complementary programming, linear programming is seen as a more convenient method for modelers because it has a simple linear objective function and does not require the strict condition of equal numbers of variables and constraint equations.

The interpretation of optimal solutions generated by solving spatial equilibrium models with assumed linear (or logarithm) supply and demand functions should consider findings of econometric research. The optimal solutions can be mathematically sound but may not have realistic economic meaning. The notable example is that the optimal price of a product may be lower than the average production cost to produce it. In addition, modelers should be careful when they report the value of the objective function. If they apply the original functions of supply and demand, they need to recalculate the total economic surplus from the estimated total loss, a relatively simple task with the assistance of command syntaxes in GAMS.

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